Pion structure functions at 2 loops: final state interaction with spectator

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Abstract

Pion structure functions are considered in the framework of QCD sum rules at the next to leading order. The 2-loop diargam, representing the gluon exchange between the struck quark and spectator, is calculated for space-like pion momentum $p^2 < 0$. After Borel transformation we obtain a leading twist correction to the pion structure function, which does not violate Bjorken scaling. Partial results can be used in the calculation of complete set of α_s corrections to the pion structure functions, which may serve as an intitial condition in the evolution equation. Probabilistic interpretation of the structure functions is discussed.

1 Introduction

The nucleon structure functions, namely, the parts of the 2-point electromagnetic current correlator

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z \, e^{iqz} \, \langle N(p)| j_\mu^{em}(z) j_\nu^{em}(0) |N(p)\rangle \tag{1}$$

are measured in the process of deep inelastic scattering. In the Bjorken limit they represent the sum of the probabilities $f_i(x)$ to find a parton i carrying the part $x = Q^2/2(pq)$ of the nucleon momentum p. This long time known statement is based on the Wilson Operator Product Expansion, which leads to the so-called twist expansion (for a review see [1]):

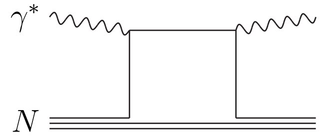
$$W_{\mu\nu} \sim \sum \frac{C}{x^J} \left(\frac{M}{Q}\right)^{d-J-2} \tag{2}$$

where $Q^2 = -q^2$, dimensionless constants C originate from local operators $O_{\mu_1 \ldots \mu_J}$ of dimension d and spin J, averaged by the nucleon state:

$$\langle N(p)| O_{\mu_1 \ldots \mu_J} |N(p)\rangle = p_{\mu_1} \ldots p_{\mu_J} M^{d-J-2} C + \ldots \tag{3}$$

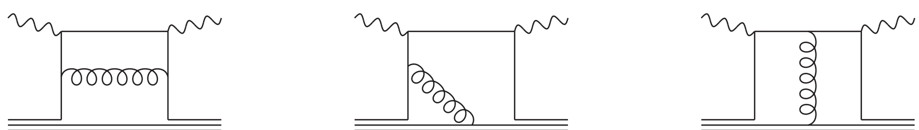
The most important are the lowest twist biquark operators $O \sim \bar{q}\gamma_\mu q$. Introducing the light-cone wave functions, one comes to a probabilistic interpretation of the nucleon structure functions at high Q^2.

The leading order contribution of the 2-quark operator is commongly visualized by the following graph:



(4)

The lower lines stand for the soft nucleon constituents (spectator), but the upper one shows hard (struck) quark, interacting with the virtual photon. QCD interaction has several topologically distinct diagrams:



a) evolution b) initial state c) final state
interaction with spectator (5)

The interaction of the struck quark with gluon field (5a) leads to renormalization of the operators (3). It yields the $\ln Q^2$ dependence of the structure functions, governed by the evolution equation. Various anomalous dimensions have been calculated at the 2-loop order [2], and the 3-loop results were published recently [3]. The diagrams (5b), (5c) with spectator involved display the contribution of the quark-quark-gluon operator. Consequently, they are either a gauge artefact [4] or higher twist corrections [5]. They do not admit any probabilistic intepretation.

However, recently a statement appeared in literature [6], that initial or final state interaction with spectator may give rise to a leading twist corrections, so the propabilistic interpretation of the structure functions should be reconsidered. In [6] it was demonstrated by a model calculation of the difractive dissociation cross section. A source of these corrections is the P-exponent between the quark fields, necessary to keep the gauge invariance of the structure functions. Although its argument vanishes in the light cone gauge $A^+ = 0$, it may generate a nonzero contribution due to additional pole $1/k^+$ of the gluon propagator here, as the authors of [6] demonstrated. For a more discussion of the P-exponent role see [7]. The initial and final state interactions were also shown [8] to give a leading-twist contribution in Drell-Yan process and semi-inclusive deep inelastic scattering.

Effect of the spectator interaction can be explicitly calculated in the framework of the sum rule approach. Here the nucleon state is approximated by some current with appropriate quantum numbers, acting on the vacuum. Then the problem is reduced to the calculation of the 4-point current correlator in vacuum. It has poles at p^2 equal to the masses of physical states (nucleons) and discontinuity at the continuum. By means of dispersion relation it can be analitically continuated to space-like region $p^2 < 0$, where it can be calculated as perturbative QCD series. Borel transformation by p^2 suppresses the contibution of the higher states and continuum and one obtains explicit expression for the structure functions in the intermediate x region, where the sum rules are supposed to work. Such a program was performed at the leading order in [9] for spin 1/2 nucleons.

The calculation of the α_s corrections to the nucleon structure functions will involve 3-loop graphs. It is simpler, however, to begin from the pion case, since it has one loop less. The

probabilistic interpetation, anomalous dimensions and the statement of [6] do not rely on the hadron structure, so they are applicable to the pion as well. The subject of this paper is the calculation of the final state interaction in the pion. In the next section we briefly review the derivation of the pion structure functions at the leading order, performed in [10], and then present original results. Technical details are dropped to Appendix.

2 Leading order pion structure functions

One considers the 4-point current correlator:

$$\Pi_{\mu\nu\alpha\beta}(p,q) \;=\; -\int dx\,dy\,dz\,e^{i(px+qy-pz)}\,\langle 0|T j^{\dagger}_{5\alpha}(x)\,j^{em}_{\mu}(y)\,j^{em}_{\nu}(0)\,j_{5\beta}(z)|0\rangle \tag{6}$$

where $j_{5\beta} = \bar{u}\gamma_\beta\gamma_5 d$ is the pion axial current, $j^{em}_\mu = \sum_q e_q \bar{q}\gamma_\mu q$ electromagnetic one. If momentum p is close to the pion mass shell $p^2 \approx m_\pi^2$, one may insert the intermediate pion state, obtaining:

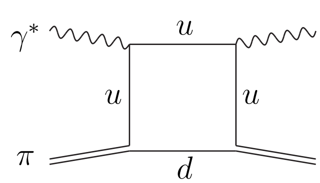
$$\Pi_{\mu\nu\alpha\beta}(p,q) \;=\; \frac{f_\pi^2\,p_\alpha p_\beta}{(p^2-m_\pi^2)^2}\int dy\,e^{iqy}\,\langle \pi(p)|T j^{em}_\mu(y)\,j^{em}_\nu(0)|\pi(p)\rangle \tag{7}$$

where f_π is the pion decay constant $\langle 0|j_{5\beta}(0)|\pi(p)\rangle = i f_\pi p_\beta$. Authors of [10] took the incoming and outgoing pions with different momenta $p_1^2 \neq p_2^2$ (but $(p_1 q) = (p_2 q)$). However, it is not necessary for our purposes: at the leading order results are equal. Moreover, the 2-loop calculation, performed in the next section, seems untractable in this case. Correlator (7) has discontinuity by the variable $s = (p+q)^2$:

$$\mathrm{Disc}_s\Pi_{\mu\nu\alpha\beta} \;\equiv\; \Pi_{\mu\nu\alpha\beta}(s+i0) \;-\; \Pi_{\mu\nu\alpha\beta}(s-i0) \;=\; 8\pi\frac{f_\pi^2\,p_\alpha p_\beta}{(p^2-m_\pi^2)^2}\,W_{\mu\nu} \tag{8}$$

The tensor $W_{\mu\nu}$ consists of the pion structure functions.

In the quark basis the correlator (6) is given by the box diagram at the leading order:



(9)

and the diagram with u, d quarks exchanged. It is easy to show, that all other diagrams (unlike the photon-photon scattering) are not essential in the Bjorken limit.

Calculation of the diagram (9) gives the following imaginary part:

$$\mathrm{Disc}_s\Pi_{\mu\nu\alpha\beta}(p,q) \;=\; -\,\frac{3e_u^2}{\pi}\,\frac{x(1-x)}{p^2}\,p_\alpha p_\beta\left(-\,e_{\mu\nu} \;+\; \frac{4x^2}{Q^2}\,\hat{P}_\mu\hat{P}_\nu\right) \;+\; h.t. \tag{10}$$

$$\text{where} \qquad e_{\mu\nu} \;=\; g_{\mu\nu} \;-\; \frac{q_\mu q_\nu}{q^2}\,, \qquad \hat{P}_\mu \;=\; p_\mu - q_\mu\frac{(pq)}{q^2}\,.$$

Letters "$h.t.$" stand for the higher twist contributions, i.e. $\sim \ln(-p^2)$, 1, p^2 etc, small with respect to the leading term $1/p^2$ in the Bjorken limit $p^2 \to 0$.

In the QCD sum rule approach one applies Borel operator to the variable $-p^2$ in order to suppress the contribution of the higher states and continuum. Then the following sum rule can be obtained:

$$\begin{aligned} \hat{\mathcal{B}}_{M^2} \mathrm{Disc}_s \Pi_{\mu\nu\alpha\beta} &= \frac{8\pi f_\pi^2}{M^4} \, e^{-m_\pi^2/M^2} \, p_\alpha p_\beta W_{\mu\nu} \\ &= \frac{3e_u^2}{\pi M^2} \, x(1-x) \; p_\alpha p_\beta \left(-\, e_{\mu\nu} \, + \, \frac{4x^2}{Q^2} \, \hat{P}_\mu \hat{P}_\nu \right) \, + \, h.t. \end{aligned} \tag{11}$$

The Borel mass should be taken [10]

$$M^2 \, = \, 8\pi f_\pi^2 \tag{12}$$

Then one extracts the u-quark distribution in the pion:

$$f_u(x) \, = \, 6x(1-x) \tag{13}$$

In fact, the sum rules are not valid for x close to 0 and 1, so exact value of M^2 as well as the normalization integral

$$\int_0^1 dx \, f_u(x) \, = \, 1 \tag{14}$$

are modified by various effects. We will not pursue this point further, see [10] for details.

The sum rule (11) demonstrates the following properties of the parton model:

- Callan-Gross relation $F_2 = 2xF_1$.
- Bjorken scaling $F(x, Q^2) = F(x)$.
- Probabilistic interpretation of the function (13) is supported by the normalization integral (14).

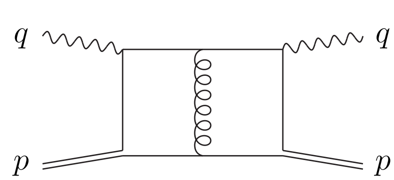
The Bjorken scaling does not survive the α_s-corrections. In particular, the leading order evolution equation gives the $\ln Q^2$ term:

$$\Delta f_{evol} \, = \, 2 \, \frac{\alpha_s}{\pi} \ln Q^2 \, [\, 1 - x \, + \, 4\,(1-x)\, x \, \ln(1-x) \, - \, 2\,(1-2x)\, x \, \ln x \,] \tag{15}$$

Vacuum condensates also have their contributions, see [10]. In this paper we will pay attention to perturbative calculation of the interaction with spectator.

3 Gluon exchange with spectator

In this section we will calculate the diagram



(16)

describing the interaction between the struck quark and spectator. To avoid extra discontinuities, the pion momentum is taken to be space-like $p^2 = -P^2 < 0$.

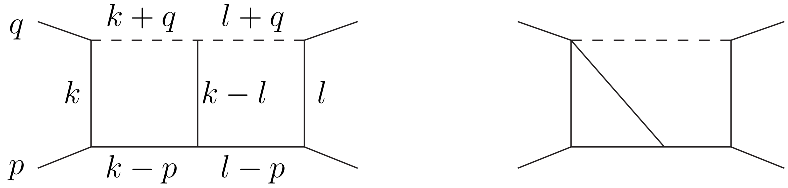
We will consider its contribution to the tensor structure $p_\mu p_\nu p_\alpha p_\beta$, i.e. to the function F_2. It it separated out by multiplying the diagram (16) on the projector:

$$\frac{q^4}{d(d-2)\Delta^4}\left[3\Delta^2\, e_{(\mu\nu}e_{\alpha\beta)} \;+\; 6(d+1)q^2\Delta\, e_{(\mu\nu}\hat{P}_\alpha\hat{P}_{\beta)} \;+\; (d+1)(d+3)q^4\,\hat{P}_\mu\hat{P}_\nu\hat{P}_\alpha\hat{P}_\beta\right]$$

where $\Delta = (pq)^2 - p^2q^2$. One can put the space-time dimention $d = 4$ from the beginning since, as explained below, only finite integrals will be important. Then the calculation is reduced to evaluation of some scalar integrals.

Such 2-loop scalar integrals were considered in the calculation of the α_s^2 anomalous dimensions of the structure functions [2]. There it was sufficient to put $p^2 = 0$ from the very beginning, since all IR poles cancel in the operator renormalization. In the sum rule approach one may not accept such simplification, since the leading order integral (10) already behaves as $1/p^2$. In dimensional regularization all the divergences have the pole $1/(d-4)$, regardless of their structure (UV or IR). For instance, it seems impossible to distinguish the terms $1/p^2$ and $\ln(-p^2)$ in such way. The first terms are interesting for us, while the later ones represent a higher twist correction. Consequently, we have to keep p^2 finite.

The calculation is simpler in the Feynman gauge, since there are only 2 graphs of the leading twist:



(17)

Indeed, consider the left graph of eq (17). Let us choose a frame $p = (0, P, 0_\perp)$ and take a part of the phase space with the integration momenta $k, l \sim p$. The volume of this region is $\int d^4k\, d^4l \;\sim p^8$. Each propagator, depicted by the solid line on the graph, brings the factor $1/p^2$ here, while the dashed line brings $1/q^2$. Consequently the left graph has the order $p^8/(p^2)^5(q^2)^2 = 1/(p^2q^2)$. The second graph of the leading twist, the right one in eq (17), is obtained by removing single dash line. (If both dash lines are removed, the integral becomes real, since no intermediate state can be inserted.) But if any solid line is removed, the twist increases and such diagram can be ignored.

However, the most complicated 2-box graph of (17) turns out to be real:

$$\mathrm{Disc}_s \int_{k,l} \frac{1}{k^2\, l^2\, (k-l)^2\, (k-p)^2\, (l-p)^2\, (k+q)^2\, (l+q)^2} \;=\; 0 \tag{18}$$

We do not know a simple way to prove this result, so we put (rather cumbersome) details of the calculation in Appendix. In the evaluation of (16) one encounters 2 integrals of the box-triangle leading twist topology:

$$J_1 \;=\; \mathrm{Disc}_s \int_{k,l} \frac{1}{k^2\, l^2\, (k-l)^2\, (k-p)^2\, (l-p)^2\, (l+q)^2} \tag{19}$$

$$J_2 \;=\; \mathrm{Disc}_s \int_{k,l} \frac{2(kq)+q^2}{k^2\, l^2\, (k-l)^2\, (k-p)^2\, (l-p)^2\, (l+q)^2} \tag{20}$$

Exact results for these integrals (arbitrary p^2, q^2, s) are available in Appendix (A21), (A22).

The contribution of the diagram (16) to the structure $p_\mu p_\nu p_\alpha p_\beta$ of the correlator (6) discontinuity is:

$$2^{11}\pi e_u^2\alpha_s\, ix(1-x)\,\big[-(1-2x)\,J_1\,+\,x\,Q^{-2}\,J_2\big]\,+\,h.t. \tag{21}$$

Substituting it into the sum rule (11), we obtain the following correction to the quark distribution function (13):

$$\begin{aligned}\Delta f(x) \;&=\; \frac{8\alpha_s}{\pi}x(1-x)\Bigg\{\,3(1-2x)\left[\mathrm{Li}_3\left(1-\frac{1}{x}\right)-\zeta_3\right]-\,(1-2x)\,\ln\left(\frac{1}{x}-1\right)\\ &\times\left[\mathrm{Li}_2\left(1-\frac{1}{x}\right)-\frac{\pi^2}{6}\right]-\,2\,\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\ln x\,\ln\left(\frac{1}{x}-1\right)-\frac{\pi^2}{6}\Bigg\}\end{aligned} \tag{22}$$

Notice, that

$$\int_0^1\Delta f(x)\,dx\;=\;0 \tag{23}$$

So this correction does not change the structure function normalization (14).

4 Conclusion

We have calculated a leading-twist α_s correction to the pion structure functions (22) due to the final state gluon exchange between the struck quark and spectator in the Feynman gauge. We certainly do not claim, that it is not cancelled by the initial state interaction or does not vanish in some specific gauge. Although the arguments, based on the operator expansion of the 2-point current product are of no doubt, they are less obvious being applied to the 4-point current correlator, considered in the sum rule approach. The spectator interaction, which occurs immediately before or after the virtual photon absorption, may give rise to a leading twist effect beyond the series (2).

The initial state diagram (5b) involves more scalar graphs, which require additional cumbersome calculations to perform. If it does not cancel the final state interaction in the sum, this will support the statement of [6], and probabilistic interpretation of the structure functions should be reconsidered. But even if it does, the calculation of the diargams (5a) with only the struck quark involved would also give a new information: its infinite part must reproduce the known $\ln Q^2$ contribution (15), determined by the evolution, while the finite part would be a new correction. These diagrams altogether will provide us with refined pion structure functions to be fitted with experimental data.

Acknowledgment

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Appendix

Below we describe the calculation of the two-loop integrals (18) and (19), (20).

Notations and conventions. Vecotors p, q are taken to be space-like, while $p+q$ is the time-like one:

$$p^2 \,=\, -P^2 \,<\, 0\,, \qquad q^2 \,=\, -Q^2 \,<\, 0\,, \qquad s \,=\, (p+q)^2 \,>\, 0\,. \tag{A1}$$

It is convenient to perform the calculation in the center of mass frame $p+q=(\sqrt{s},0)$. One may choose

$$p \,=\, (\sqrt{\omega^2-P^2},0,0,\omega)\,, \qquad q \,=\, (\sqrt{\omega^2-Q^2},0,0,-\omega) \tag{A2}$$

where

$$\omega = \sqrt{\frac{\nu^2-P^2Q^2}{s}}\,, \qquad \nu=(pq)\,. \tag{A3}$$

In the calculation of the 2-particle cut we use the 2-particle phase volume:

$$\begin{aligned} d\Gamma_2 \;&\equiv\; \int \frac{d^3k}{(2\pi)^3 2k^0}\,\frac{d^3k_2}{(2\pi)^3 2k_2^0}\,(2\pi)^4\delta^4(p+q-k-k_2) \\ &=\; \frac{1}{16\pi\,\omega\sqrt{s}}\int_{T_-}^{T_+} dT \end{aligned} \tag{A4}$$

where $k^2=k_2^2=0$ and

$$T \,=\, -(p-k)^2\,, \qquad T_\pm \,=\, \nu\pm\omega\sqrt{s} \tag{A5}$$

In (A4) we integrated out azimuthal angle ϕ in xy plane, since in the frame (A2) the amplitudes do not depend of it.

The 3-particle phase volume is:

$$\begin{aligned} d\Gamma_3 \;&=\; \int \frac{d^3k_1}{(2\pi)^3\,2k_1^0}\,\frac{d^3k_2}{(2\pi)^3 2k_2^0}\,\frac{d^3k_3}{(2\pi)^3\,2k_3^0}\,(2\pi)^4\,\delta^4(p+q-k_1-k_2-k_3) \\ &=\; \frac{1}{(2\pi)^5}\int \frac{d^3k_1}{2k_1^0}\,\frac{d^3k_2}{2k_2^0}\,\delta[(p+q-k_1-k_2)^2-\lambda^2]\,\theta(p^0+q^0-k_1^0-k_2^0) \end{aligned} \tag{A6}$$

where $k_1^2=k_2^2=0$, but $k_3^2=\lambda^2$, the mass λ is necessary to regularize the IR divergencies. In spherical coordinates

$$\begin{aligned} d\Gamma_3 \;&=\; \frac{1}{(2\pi)^5}\cdot\frac{1}{8}\int_0^{\sqrt{s}} dk_1 \int_0^{\sqrt{s}-k_1} dk_2 \int_0^{\pi} d\theta_1 \int_0^{\pi} d\theta_2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \\ &\quad\times\delta\left[\frac{\cos\alpha-\cos\theta_1\cos\theta_2}{\sin\theta_1\sin\theta_2} \;-\; \cos(\phi_1-\phi_2)\right] \end{aligned} \tag{A7}$$

where we denoted

$$\cos\alpha \;=\; \frac{s-2\sqrt{s}(k_1+k_2)+2k_1k_2-\lambda^2}{2k_1k_2}\,.$$

For a given θ_1 the condition $\cos^2(\phi_1 - \phi_2) < 1$ sets the following restriction on the integration range:

$$|\cos\alpha| < 1\,, \qquad \cos(\theta_1 + \alpha) < \cos\theta_2 < \cos(\theta_1 - \alpha)$$

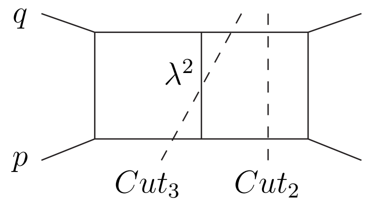
It is convenient to introduce new variable ξ instead of θ_2 and $t_{1,2}$ instead of $k_{1,2}$:

$$\cos\theta_2 \;=\; \cos\theta_1\cos\alpha + \sin\theta_1\sin\alpha\cos\xi\,, \qquad k_i \;=\; \frac{\sqrt{s}}{2}(1 - t_i) \tag{A8}$$

The δ-function is excluded by integrating out ϕ_2. Since the amplitudes are independent of the angle ϕ_1, we integrate it also. Then the 3-particle phase volume (A7) takes the most convenient form:

$$d\Gamma_3 \;=\; \frac{1}{(2\pi)^4}\cdot\frac{s}{16}\int_{\lambda^2/s}^{1} dt_1 \int_{\lambda^2/(st_1)}^{1-t_1+\lambda^2/s} dt_2 \int_0^{\pi}\sin\theta_1\,d\theta_1 \int_0^{\pi} d\xi \tag{A9}$$

2-box diagram consists of two cuts



(A10)

and symmetric ones. Since all internal lines are massless, both cuts are IR divergent. A common way to regularize it is the introduction of small "photon" mass λ^2. To evaluate the 2-particle cut, one needs to know the 1-loop box integral:

$$I_\Box \;=\; \int_l \frac{1}{l^2\,(l-p)^2\,(l+q)^2\,[(l-p+k)^2 - \lambda^2]} \tag{A11}$$

The Feynman representation of the propagator product yelds the following 3-fold integral:

$$\begin{aligned} I_\Box \;&=\; \frac{i}{16\pi^2}\int_0^1 dx \int_0^x dy \int_0^y dz\,\times \\ &\times \left[Q^2(x-y)(1-x) + Tz(1-x) + P^2(y-z)(1-x) - s(x-y)(y-z) + \lambda^2 z\right]^{-2} \end{aligned} \tag{A12}$$

This integral is real for $s < 0$, since the denominator is never zero in this case. So we calculate it for negative s and then analytically continuate to positive region. The result is:

$$I_\Box = -\frac{i}{16\pi^2 sT}\left[\mathrm{Li}_2\left(1 - \frac{T}{P^2}\right) + \mathrm{Li}_2\left(1 - \frac{T}{Q^2}\right) + \frac{1}{2}\ln^2\frac{-sT^2}{\lambda^2 P^2 Q^2} + \frac{\pi^2}{3}\right] \tag{A13}$$

Then one calculates the contribution of both 2-particle cuts:

$$\begin{aligned} 2\,Cut_2 \;&=\; \int d\Gamma_2 \frac{i}{(p-k)^2}\left\{ I_\Box|_{s+i0} + I_\Box|_{s-i0}\right\} \\ &=\; -\frac{1}{128\pi^3 sP^2Q^2}\left\{\ln^2\frac{\lambda^2}{s} - 4\ln\frac{\lambda^2}{s} + 8 - \frac{\pi^2}{3} + 3\ln^2\frac{T_-}{PQ} - \ln^2\frac{P}{Q}\right. \\ &\quad + \frac{4\nu}{\omega\sqrt{s}}\ln\frac{T_-}{PQ}\left(2 - \ln\frac{\lambda^2}{s}\right) - \frac{P^2 - Q^2}{\omega\sqrt{s}}\ln\frac{P}{Q}\ln\frac{T_-}{PQ} \\ &\quad \left. + \frac{\sqrt{s}}{\omega}\left[\mathrm{Li}_2\left(1 - \frac{P^2}{T_-}\right) + \mathrm{Li}_2\left(1 - \frac{Q^2}{T_-}\right) + \frac{1}{2}\ln^2\frac{T_-}{PQ} + \frac{1}{2}\ln^2\frac{P}{Q}\right]\right\} \end{aligned} \tag{A14}$$

The 3-particle cut is given by the integral:

$$Cut_3 = \int \frac{d\Gamma_3}{(p+q-k_1)^2(p+q-k_2)^2(p-k_1)^2(q-k_2)^2} \tag{A15}$$
$$= \int \frac{d\Gamma_3\,(s-2\sqrt{s}k_1)^{-1}(s-2\sqrt{s}k_2)^{-1}}{[-P^2-2k_1(\sqrt{\omega^2-P^2}-\omega\cos\theta_1)][-Q^2-2k_2(\sqrt{\omega^2-Q^2}+\omega\cos\theta_2)]}$$

Integration over the angles yields:

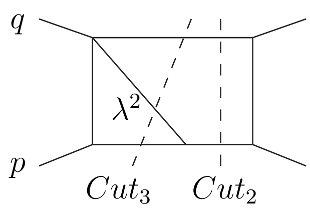
$$Cut_3 = \frac{1}{64\pi^3 s}\cdot\frac{1}{4\omega\sqrt{s}}\int_{\lambda^2/s}^{1}\frac{dt_1}{t_1}\int_{\lambda^2/(st_1)}^{1-t_1+\lambda^2/s}\frac{dt_2}{t_2} \tag{A16}$$
$$\times\frac{1}{st_1t_2+P^2t_1+Q^2t_2}\ln\frac{(1-t_1-t_2)P^2Q^2+T_+(st_1t_2+P^2t_1+Q^2t_2)}{(1-t_1-t_2)P^2Q^2+T_-(st_1t_2+P^2t_1+Q^2t_2)}$$

This integral is IR divergent at $t_{1,2}\to 0$ (or $k_{1,2}\to\frac{\sqrt{s}}{2}$). The expresion in the second line of (A16) is regular there, so we put $\lambda^2=0$ everywhere except the integration limits; account of the terms neglected would lead to power corrections in λ, but not logarithmic ones. Evaluation of (A16) yields exactly $-1/2$ of equation (A14), so

$$2\,Cut_2 + 2\,Cut_3 = 0$$

which proves the equation (18).

Box-triangle diagram



(A17)

is calculated in similar way. The triangle 1-loop integral is:

$$I_\Delta = \int_l \frac{1}{l^2\,(l+p)^2\,[(l+k)^2-\lambda^2]}$$
$$= -\frac{i}{16\pi^2(T-P^2)}\left[\mathrm{Li}_2\left(1-\frac{P^2}{T}\right)+\ln\frac{T}{\lambda^2}\ln\frac{T}{P^2}\right] \tag{A18}$$

The 2-particle cut is equal to:

$$Cut_2 = \int d\Gamma_2\frac{i}{(k-p)^2}\,i^3\,I_\Delta$$
$$= \frac{i}{256\pi^3\omega\sqrt{s}P^2}\Big[2\,\mathrm{Li}_3(1-y)\,+\,3\,\mathrm{Li}_3(1-1/y)\,-\,2\,\ln y\,\mathrm{Li}_2(1-y)$$
$$-\frac{2}{3}\,\ln^3 y\,+\ln\frac{P^2}{\lambda^2}\,\mathrm{Li}_2(1-1/y)\Big]\Big|_{y_-}^{y_+}\,,\qquad y=\frac{T}{P^2} \tag{A19}$$

The 3-particle cut is given by the same integral as (A16), but without t_1 in denominator:

$$
\begin{aligned}
Cut_3 &= \int \frac{i^3\, d\Gamma_3}{(p+q-k_2)^2\,(p-k_1)^2\,(q-k_2)^2} \\
&= \frac{i}{256\pi^3\omega\sqrt{s}P^2}\left\{\left[\ln\frac{y_-}{y_+} - \ln\frac{s}{\lambda^2}\right]\mathrm{Li}_2(1-1/y_+)\right. \\
&\quad \left. -\,2\,\mathrm{Li}_3(1-y_+) \,-\, \frac{1}{3}\ln^3 y_+ \,-\, (y_- \leftrightarrow y_+)\right\} \qquad \text{(A20)}
\end{aligned}
$$

In the calculation we used the following properties of the function Li_3:

$$
\mathrm{Li}_3(1-z) + \mathrm{Li}_3(1-1/z) + \mathrm{Li}_3(1/z) = \zeta_3 - \frac{\pi^2}{6}\ln z + \frac{1}{3}\ln^3 z - \frac{1}{2}\ln(z-1)\,\ln^2 z
$$

$$
\mathrm{Li}_3(-z) \,-\, \mathrm{Li}_3(-1/z) \,=\, -\frac{\pi^2}{6}\ln z \,-\, \frac{1}{6}\ln^3 z
$$

Summing up (A19) and (A20), one obtains the integral (19):

$$
\begin{aligned}
J_1 &= -\frac{i}{256\pi^3\omega\sqrt{s}P^2}\left[3\,\mathrm{Li}_3\left(1-\frac{P^2}{T_+}\right) \,-\, 3\,\mathrm{Li}_3\left(1-\frac{P^2}{T_-}\right)\right. \\
&\quad \left. +\,\ln\frac{Q^2}{s}\,\mathrm{Li}_2\left(1-\frac{P^2}{T_+}\right) \,-\, \ln\frac{Q^2}{s}\,\mathrm{Li}_2\left(1-\frac{P^2}{T_-}\right)\right] \qquad \text{(A21)}
\end{aligned}
$$

The intergal (20) is calculated in similar way:

$$
\begin{aligned}
J_2 &= -\frac{i}{256\pi^3\omega\sqrt{s}P^2}\left[(T_- - 2T_+)\,\mathrm{Li}_2\left(1-\frac{P^2}{T_-}\right) \,-\, (T_+ - 2T_-)\,\mathrm{Li}_2\left(1-\frac{P^2}{T_+}\right)\right. \\
&\quad \left. -\,T_+\ln\frac{T_-}{P^2}\ln\frac{Q^2}{s} \,+\, T_-\ln\frac{T_+}{P^2}\ln\frac{Q^2}{s}\right] \qquad \text{(A22)}
\end{aligned}
$$

In the Bjorken limit $P^2 \to 0$ and

$$
\omega\sqrt{s} \approx \nu = \frac{Q^2}{2x}\,, \qquad s \approx Q^2\frac{1-x}{x}\,, \qquad T_+ \approx \frac{Q^2}{x}\,, \qquad T_- \approx xP^2\,. \qquad \text{(A23)}
$$

Notice cancellation of the terms $\sim \ln^n P^2/P^2$ in (A21), (A22); they would violate Bjorken scaling.

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